Design Data for a Disk Rotating in a Rarefied Environment

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SOLUTIONS of the Navier-Stokes and energy equations with the slip-velocity and temperature-jump boundary conditions for a free, infinite, isothermal disk rotating in a variable property, rarefied fluid have resulted in realistic engineering design formulations for tangential moment-coefficients, radial shear-stress coefficients, and Nusselt numbers as functions of rarefaction.

Introduction

Rotating disks are encountered in many practical situations.[1] However, existing formulations for estimating the transport parameters for a disk rotating in rarefied environments are currently based upon a first-order perturbation-analysis for the tangential moment-coefficient and Nusselt numbers.[2] Variations in the wall boundary conditions with rarefaction are ignored in such an analysis, and, consequently, the predicted tangential moment-coefficient is identical to the results obtained from continuum theory. Moreover, existing results obtained for Nusselt numbers associated with an isothermal, infinite disk rotating steadily in a slightly rarefied gas indicate that to first approximation the heat transfer is unaffected whenever the slip-velocity coefficient is equal to the temperature-jump coefficient. However, experimental studies employing finite disk arrangements indicate typical deviations in these parameters of the order of 500% from the continuum results at a Knudsen number of 0.2.[3,4] Moreover, Barbee and Shih,[5] using a Karman-Pohlhausen integral technique with a free-disk geometry, found these transport parameters to differ markedly from continuum theory predictions, but simplifying assumptions led to uncertainties in the analytical model.

Thus, in the absence of experimental data, it was felt that a more penetrating analytical study should be made to provide more precise engineering design formulations for the various moment and heat-transfer coefficients.

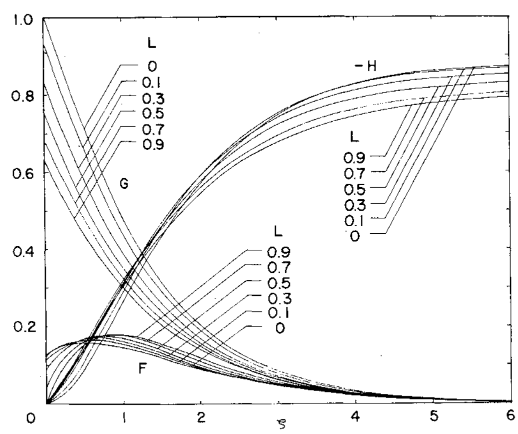

Fig. 1 Nondimensional velocity distributions for various conditions of slip.

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Formulation

Employing the physical model and nomenclature of Ref. 6, envoking the slip-velocity and temperature-jump boundary conditions of Ref. 3, making the additional substitution that

$$L = \left[\frac{\Omega}{\nu_\infty C}\right]^{1/2} \rho l_o \tag{1}$$

where l_o and L are the mean-free-path of the gas at the disk surface and the nondimensional-mean-free-path, respectively, assuming a linear relationship between both the viscosity and thermal conductivity and the temperature, and neglecting dissipation, the transfer equations and their corresponding boundary conditions become

$$H' + 2F = 0; \; H(0) = 0, H(\infty) = -\text{const} \tag{2}$$

$$F^2 + HF' - G^2 = F''; \; F(0) = LF'(0), F(\infty) = 0 \tag{3}$$

$$2FG + GH' = G''; \; G(0) = 1 + LG'(0), G(\infty) = 0 \tag{4}$$

$$PrHS' = S''; \; S(0) = 1 + \kappa LS'(0), S(\infty) = S'(\infty) = S''(\infty) = 0 \tag{5}$$

where $\kappa = 75\pi/128$.

Solution

The details of the Newton-Cotes, five-point, forward integration method used for integrating the nonlinear, ordinary differential equations (Eq. 2–5) and the corresponding convergence criteria are discussed in detail in Ref. 7.

Results

The results are presented in Figs. 1–3. Tangential, radial, and axial velocity profiles are presented in Fig. 1 for air with variable fluid properties for different conditions of rarefaction. Variable fluid property temperature profiles and the variation in the pressure field for air with constant fluid properties and $Pr = 0.72$ are presented in Fig. 2, where P_o is the value of the nondimensional pressure at the disk. The tangential moment-coefficient, $C_m = -2\pi C^{1/2}G'(0)/Re^{1/2}$, the radial shear-stress-coefficient, $C_r = 2C^{1/2}F'(0)/Re^{1/2}$, and the Nusselt number, $Nu = -C^{1/2}S'(0)/Re^{1/2}$, are depicted in Fig. 3 for varying rarefactions and Prandtl numbers.

Also presented in Fig. 3 is a comparison between both the tangential moment-coefficient and Nusselt number of Ref. 5 and the corresponding coefficients of this study. Significant disagreement is noted in the moment-coefficients. This discrepancy results from the approximations associated

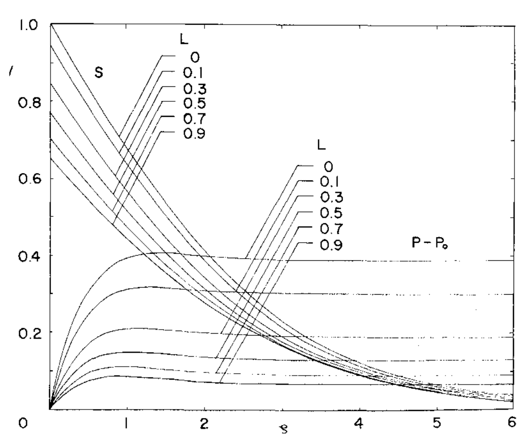

Fig. 2 Nondimensional temperature and pressure distributions for various conditions of slip.

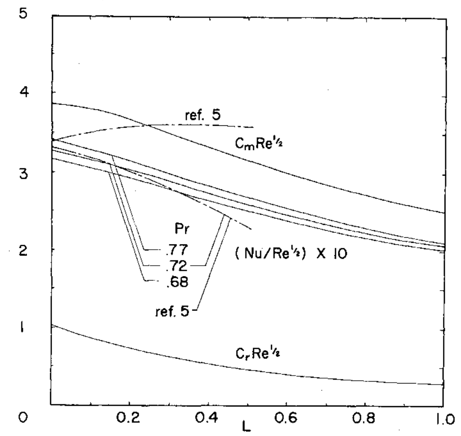

Fig. 3 Drag, friction moment, and heat-transfer coefficients for various conditions of slip.

with the Karman-Pohlhausen method. Cochran[8] demonstrated that, for a continuum situation, the magnitude of the tangential moment-coefficient increased from 3.37 to 3.87 when this restriction is relaxed, which further substantiates the validity of this work (see Fig. 3).

In contrast to the results of the first-order perturbation-analysis of Shidlovskiy as well as the predictions of continuum theory, the results of this study indicate that all of the transport characteristics decrease significantly with increasing rarefaction as should be expected since increasing rarefaction decreases the frequency of molecular interaction at solid boundaries.

References

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Parachute Critical Opening Velocity

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Nomenclature

a_r = radial (outward) acceleration of canopy element
D_0 = parachute constructed diameter
f_i = functional relationship (i = 1,2,3)
g = acceleration of gravity
l_s = suspension line length
N = number of suspension lines
v_c = parachute critical opening velocity
γ_{avg} = canopy material average mass per unit area
ρ = atmosphere mass density

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Introduction

PARACHUTE critical opening velocity v_c, also called the squidding velocity, is discussed by O'Hara[1] and Brown.[2] Experimental data presented by Brown[2] show that v_c is a function of several geometrical factors (N, l_s/D_0, porosity), as well as of parachute size. In this Note, a rudimentary analysis is used to obtain dimensionless products associated with v_c scale factor (i.e., size) effects.

Analysis

Squidding is essentially a stall or hold in the normal parachute inflation process which occurs at or near the end of the initial phase of inflation.[3,4] The squidded parachute canopy, shown schematically in Fig. 1, is of small cross section and approximately cylindrical. If one considers only geometrically similar parachutes in incompressible flow, the forces per unit area tending to expand and/or collapse the canopy radially are a function of the pressure difference across the canopy, which is in turn a function of $\frac{1}{2}\rho v$.[2] The effective canopy mass per unit area is a function of γ_{avg}. Thus, the outward acceleration of a unit area of canopy is, in dimensionless form,

$$a_r/g = f_1(\tfrac{1}{2}\rho v^2)/gf_2(\gamma_{avg}) \quad (1)$$

The squidding parachute represents an equilibrium condition with $a_r = 0$ and $v = v_c$. These conditions and Eq. (1) suggest the relationship

$$v_c^2/gD_0 = f_3(\gamma_{avg}/\rho D_0) \quad (2)$$

Table 1 Summary of v_c data

Source	Parachute characteristics[a]: Geometry and mass	D_0, ft	Squid conditions: v_c, fps	ρ, slug/ft³
Brown[2]	porosity = 43 ft³/ft²-sec at 10 in. water	3	95	0.00238[b]
	l_s/D_0 = 0.67	4½	87	
	N = 12	6	58	
	$g\gamma_{avg}$ = 1.6 oz/yd²[b]	9	65	
		12	50	
Berndt[3]	porosity = 100 ft³/ft²-min at ½ in. water	28	331	0.00159
	l_s/D_0 = 1.0	28	351	0.00159
	N = 28	28	356	0.00159
	$g\gamma_{avg}$ = 1.1 oz/yd²	28	334	0.00124

[a] All chutes are solid flat circular type.
[b] Assumed value.

Discussion

The only experimental data which appear available to check the validity of Eq. (2) are summarized in Table 1.

Fig. 1 Squidding parachute.